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THIRD SUMMER MEETING OF THE MATHEMATICAL ASSOCIATION OF AMERICA.

The Association held its third summer meeting by invitation of Dartmouth College at Hanover, N. H., on Thursday, Friday and Saturday, September 5–7, 1918, in conjunction with and following the summer meeting of the American Mathematical Society. Seventy-one persons attended the sessions, including the following 42 members of the Association:

H. L. AGARD, Williams College.
R. C. ARCHIBALD, Brown University.
R. D. BEETLE, Dartmouth College.
G. D. BIRKHOFF, Harvard University.
DANIEL BUCHANAN, Queen's University.
W. D. CAIRNS, Oberlin College.
W. B. CARVER, Cornell University.
JULIA T. COLPITTS, Iowa State College.
LENNIE P. COPELAND, Wellesley College.
LOUISE D. CUMMINGS, Vassar College.
C. H. CURRIER, Brown University.
E. L. DODD, University of Texas.
C. H. FORSYTH, Dartmouth College.
A. S. GALE, University of Rochester.
O. E. GLENN, University of Pennsylvania.
C. F. GUMMER, Queen's University.
J. G. HARDY, Williams College.
E. V. HUNTINGTON, Harvard University.
W. W. JOHNSON, U. S. Navy.
FLORENCE P. LEWIS, Goucher College.
J. MATHESON, Queen's University.
HELEN A. MERRILL, Wellesley College.
F. M. MORGAN, Dartmouth College.
G. D. OLDS, Amherst College.
H. L. OLSON, New Hampshire College.
F. W. OWENS, Cornell University.
ANNA H. PALMIÉ, College for Women, Western Reserve University.
A. D. PITCHER, Adelbert College.
JESSIE G. QUIGLEY, College of Saint Teresa.
L. H. RICE, Tufts College.
R. G. D. RICHARDSON, Brown University.
E. D. ROE, JR., Syracuse University.
CLARA E. SMITH, Wellesley College.
SARAH E. SMITH, Mount Holyoke College.
H. W. TYLER, Massachusetts Institute of Technology.
OSWALD VEBLEN, Princeton University.
C. A. WALDO, Washington University.
A. G. WEBSTER, Clark University.
J. K. WHITTEMORE, Yale University.
C. B. WILLIAMS, Kalamazoo College.
F. N. WILLSON, Princeton University.
J. W. YOUNG, Dartmouth College.

The increase in railroad rates had its very evident effect in the somewhat reduced attendance as compared with the preceding summer; there were present

nevertheless as members from greater distances—three each from Canada and Ohio, and one each from Iowa, Maryland, Michigan, Minnesota, New Jersey, Pennsylvania and Texas. The presence of Majors Huntington and Veblen, and of Professor Tyler from their governmental activities in Washington, D. C., and Maryland gave a strong suggestion of the unusual times in which we are and brought great aid in the important consultations and results which formed a unique feature of the meeting.[1]

Members and other visitors were comfortably housed in South Fayerweather Hall and College Hall. The latter afforded also a dining room and commodious social rooms which were freely placed at the disposal of all. A leaflet of general information printed by Dartmouth College especially for these meetings was of definite assistance to all. Small parties made brief excursions about the historic village and vicinity. In all respects this was a most pleasant and successful meeting. A unanimous rising vote at the joint session of the two organizations recognized the admirable character of the local arrangements and the warm hospitality shown by the Hanover faculty.

The sessions were preceded by the joint dinner of the Association with the American Mathematical Society on Thursday evening. Fifty-six were present on this occasion, which combined with its pleasant social feature an atmosphere of earnestness in the face of war times and war problems. President Huntington called upon several persons, including Dean Laycock of Dartmouth College, who welcomed the assembled guests to Dartmouth and brought fresh reports from the Plattsburg conference on military education in the colleges; Major Veblen, who urged the call of the ordnance department for able mathematicians to assist in that department, men who are capable of assuming responsibility for independent work in ballistics, as well as those not so fully able to work independently; Secretary Cairns, who read abstracts from the government's circular on the new courses to be instituted in the colleges and universities, and called attention to certain proposed war courses in mathematics which, already sketched out by a few mathematicians, should be perfected at this time; Professor Richardson, who described the instruction in navigation that has already been given in two or three universities and urged the great importance of preparing young men whose choice is the navy; Professor Willson, who at the suggestion of Major Huntington emphasized the place of descriptive geometry as one of the essential courses in the new war mathematics; Secretary Cole, who announced the publication of the first part of the Cambridge Colloquium, that by Professor G. C. Evans; Professor Webster, who gave an engrossing account of the experiences of a civilian enlisted in governmental service; and Dean Olds, who supplemented the report concerning the new war courses.

Sessions of the Association were held as announced on Friday morning and afternoon and on Saturday morning. The joint session of the Association with the Society at half past ten o'clock Friday morning, Professor G. D. Birkhoff pre-

[1] The Secretary-Treasurer regrets that, due to a mistake made by the printer, the members in several states did not receive copies of the program.

siding, was addressed by Professor A. G. Webster on "Mathematics of Warfare." As this well known mathematical physicist is on the U. S. Naval Consulting Board, he was well equipped to tell of current developments in the theory of ballistics. He described the elementary principles of ballistics for long range trajectories, summarizing the contributions to the subject of such men as Danforth, Siacci and Chapel. After developing the "ballistic function" and its differential equation, he closed by touching more briefly on the outstanding problems of interior ballistics. The address when published in its full form will furnish a notable chapter in this theory.

A valuable and suggestive program was arranged by the program committee under the chairmanship of Professor Archibald. A still greater interest and concentration of effort centered about the almost continuous conferences over the formulation of war courses in mathematics. Before this report appears, all institutions concerned will have been informed as to the United States government's recommendations or instructions in this respect. It will however remain as a unique feature of this meeting that thirty or forty professors in collegiate mathematics from representative schools in all parts of the country except the far west met in an informal capacity and after much discussion and earnest consideration came to a united view as to the character and contents of three twelve-week terms for the general military student in college, two twelve-week terms in navigation, and an eight-week course in descriptive geometry, besides suggestions as to further second year courses, so far as there will be men to attend these. The courses as formulated were transmitted through Major Huntington as recommendations made by this unofficial conference to the educational committee of the war department and will doubtless have been incorporated in whole or in part into the suggestions issued by that department and already familiar to the readers of the MONTHLY before the appearance of this report.

ORDER OF TOPICS ON THE SEPARATE PROGRAM.

(1) "The Teaching of Curve Tracing." PROFESSOR F. W. OWENS, Cornell University.

(2) "A Formula in Combinatorial Analysis." PROFESSOR J. W. YOUNG, Dartmouth College.

(3) "Trigonometric Functions—of what?" PROFESSOR W. B. CARVER, Cornell University.

(4) "Firing Data at Yale." J. K. WHITTEMORE, Yale University.

(5) "A Combined Course in Mathematics for College Freshmen." PROFESSOR A. S. GALE, University of Rochester.

(6) Report of the Committee on Mathematical Requirements. PROFESSOR J. W. YOUNG, Chairman.

(7) Presidential Retiring Address: "Plans for a History of Mathematics of the Nineteenth Century." PROFESSOR FLORIAN CAJORI, University of California.

(8) "Some Experiments in the Teaching of Descriptive Geometry." PROFESSOR F. L. KENNEDY, Harvard University.

Abstracts, numbered to correspond with the numbers on the foregoing program, are printed below, together with reports of some further informal discussions.

ABSTRACTS OF PAPERS.

(1) Professor Owens in his paper on the "Teaching of Curve Tracing" urged that the light thrown upon the whole notion of functionality by the introduction of the graph may be much intensified by a much greater use of graphical methods in the *construction* of the curve which exhibits the function. He exhibited some simple methods which he has found to be particularly helpful to students in constructing graphs for functions which are or may be given explicitly, and which make especially clear the genesis of the more complicated function from more simple ones by processes indicated in the expression for the function.

(2) Professor Young considers a class $K = (k_1, k_2, \ldots, k_n)$ on N objects. Any combination C_s of s elements of K, together with the combination $\overline{C}_s$ of the remaining $N - s$ objects is called a *partition* of K, C_s and $\overline{C}_s$ being called the *sides* of the partition. A pair, triple, . . . , i-ad of elements of K is said to occur in a partition if the pair, triple, . . . , i-ad occurs in either side of the partition. A *system* S_m of partitions $(C_{s_i}, \overline{C}_{s_i})$ $(i = 1, 2, \ldots, m)$ consist of any set of m partitions; the s_i are not assumed to be all equal, though they may be. The symbol ξ_i is used to represent any i-ad of K; P_{ξ_i} to represent the number of times the i-ad ξ_i occurs in a given S_m. Thus $P_{k_1k_2}$ means the number of times the pair $k_1\ k_2$ occurs in S_m; $P_{k_j\xi_i}$ the number of times the $(i+1)$-ad $k_j\xi_i$ occurs in S_m. The formula mentioned in the title is as follows:

$$\sum_{j\alpha_{i-1}} [P_{k_j\xi_{i-1}} - P_{k_i\xi_{i-2}} + P_{k_j\xi_{i-3}} - \ldots \pm P_{k_j\xi_1}] = (-1)_i[m + P\alpha_i - P_j\alpha_{i-1}],$$

where α_i is any given set, $k_1, k_2, \ldots, k_i$, of K, when k_j is any element of α_i, and ${}_j\alpha_{i-1}$ represents the set of $i-1$ elements obtained by removing k_j from α_i. The summation is extended over all $\xi_{i-1}, \xi_{i-2}, \ldots$ contained in ${}_j\alpha_{i-1}$. This formula may readily be summed from $j = 1$ to $j = i$, yielding a second formula.

The above formula for $i = 3$ gives

$$P_{ij} + P_{ik} + P_{jk} = m + 2\,P_{ijk}.$$

In any system S_m, in which all pairs occur the same number of times, all triples will also occur the same number of times. This theorem is readily generalized. Application of the formula is made to certain problems of arrangement —in particular to certain problems connected with a certain type of whist tournaments.

(3) Professor Carver's paper called attention to the difficulty caused for the student by the two different notions of the trigonometric functions, as functions of *angles* and functions of *numbers*. It was suggested that the student might

be helped (1) in trigonometry, by the early introduction, and *frequent use in problems*, of the radian unit; (2) in analytic geometry, by insistence upon the use of the radian unit in the plotting of such curves as $y = \sin x$ and $\rho = a\theta$; (3) in calculus, by a clear presentation of the notion of trigonometric functions of *numbers* just before the development of their derivatives.

(4) Mr. Whittemore's paper, which was an account of the course in Artillery Firing given the past year at Yale University as a three-hour course for a semester, has already appeared in this MONTHLY. It was a very timely address and in its full form offers a great aid to those who have been planning similar courses.

(5) A first step toward a combined course for freshmen was taken at the University of Rochester by Professor Gale in the year 1907–1908, when it was decided to emphasize the graphical significance of the functions occurring in the subjects taught, namely, solid geometry, trigonometry and advanced algebra. Soon thereafter considerable interweaving of these subjects was done. In 1913–1914 the calculus was introduced and a combined course, developed by Professors Gale and Watkeys, has been taught for the past four years.

The central theme of the year's work is the study of the elementary functions, algebraic and transcendental. An attempt has been made to keep in very close contact with applications of mathematics, and to attach greater importance to ideas and the development of the power to think than to purely manipulative processes. An analysis of the properties of functions and graphs consists of relations between a single function and its graph, and of the relations between various pairs of functions and their graphs. These principles may be introduced in connection with simple algebraic functions, and utilized in the presentation of the transcendental functions. Interpretation of the graphs of important functions furnishes the means for organizing and remembering various properties of the functions.

Among the applications, emphasis is laid on the determination of a function whose table of values agrees reasonably well with a given table obtained empirically, the general problem being given as a part of the method of discovery in science.

It is believed that the moulding of the subject matter into a coherent year's work gives the student a better comprehension of what mathematics is and a greater facility in its use, and that the freshmen are more interested in their work than formerly.

Among those who took part in the discussion, Professor Richardson maintained that different courses should be given according as the students do or do not expect to continue the study of mathematics, Professor Williams told of his success in combined courses for freshmen, and Professor Olds recalled the interest which was kindled by the introduction of calculus into the freshman year at Amherst College years ago.

(6) Professor Young made a report of the progress being made along the lines of the committee's activities. (See report in the MONTHLY for December, 1917, page 463, for a statement of these.) Miss Blair's report has been postponed in

its printing in order to assimilate with this report certain material which has come in in connection with two or three recent articles on formal discipline and the transfer of training.

The survey in charge of Professor G. W. Evans of courses in algebra in secondary schools is being correlated with the work of committees appointed by the associations affiliated with the Association in this work. Professor Crathorne has made a preliminary report and is now engaged in studying the results of questionnaires.

It seems altogether probable that within the next few months the committee will be able to present a report embodying definite principles, the report to be submitted to the Association and the affiliated bodies for their discussion and promulgation if the report is adopted.

(7) The address of ex-President Cajori has already been printed in *Science*. The removal of residence of Professor Cajori to California rendered his presence at the meeting impossible, and in his absence the paper was read by Professor Olds.

(8) Professor Kennedy showed by clear and excellent charts how he meets the difficulties of the students as they begin the study of descriptive geometry. He uses a set of large scale diagrams carefully drawn on heavy board in various colors, blue print and problem sheets 8″ by $10\frac{1}{2}$″ issued to the pupils, as well as tests on sheets of the same or double size, such tests of varying length being used once a week to secure concentration of effort and a rating of the student's work.

No text is used in the course. A preliminary sheet shows the method of representing geometrical figures by means of three reference planes (frame of reference) and the three working rules of orthographie projection, *e. g.*, vertical and horizontal projections lie on the same vertical line. Blue prints give successively symbols and conventions, simple and clear perspective and orthographic drawings of the elementary theorems of points and lines, planes and traces, lines and planes (*e. g.*, point of intersection of a given line and a given plane), angle between two planes, the gradual approach by rotations to axonometric and isometric projection, a plate of "facts and pitfalls" (correct and incorrect constructions side by side) mimeographed in blue and red respectively.

The large scale wall charts previously mentioned illustrate the gradual development of a problem (*e. g.*, the problem of passing a plane through three given points was shown in four stages, besides a perspective drawing) and afforded an accurate and neat drawing not feasible in quick blackboard sketching. Colors are freely used to distinguish different situations, as for example given, construction, and required lines. Some of the charts have somewhat the nature of dissolving views, in that lines, etc., not longer of importance are drawn more lightly or are omitted in the later stages.

A complete and concise notation is used both in assignment of problems and in the student's analysis of these; thus, the statement "Given a line ab and a point p, to pass a plane M through p perpendicular to ab" is given in the following form:

$$\frac{\overline{ab} \text{ and } p}{M \to p \perp \overline{\overline{ab}}}$$

The student is expected to note plainly the definite steps in his solution by the use of this notation and other means; for example, an arrow denotes the direction in which he revolves a line. Moreover this notation in connection with given lines, points, etc., mimeographed on problem sheets enables him quickly to attack the assigned problems each day. So far as shades and shadows are taught, a very few charts are used instead of a text, all oral explanation being obviated by the use on these charts of the abbreviated notation already familiar to the class.

Papers are marked with respect to accuracy, clearness and workmanship. Only limited stress is laid on originality since this is apt to develop into eccentricity. Ink is used sparingly and always where it will do the most good.

Professor Kennedy showed a plate embodying a ten minute exercise, a mid-year examination, and a plate of four problems comprising an "inverted test," *i. e.*, four problems were drawn incorrectly, the student being given fifteen minutes to pick out the errors, indicate and correct these.

Dean Randall, who was to have discussed this paper, was unable to be present by reason of the unexpected pressure of duties in inaugurating the new war courses. Professor Willson showed a copy of Monge's *Géométrie Descriptive*, published in 1799 during the French Revolution and dated according to the calendar invented for the new regime by the author and a collaborator; since the use of this calendar ceased soon thereafter this is a very unique book, being the only descriptive geometry and one of the very few books which bear this system of dating.

Meeting of the Council of the Association.

At the meeting of the Council on Friday evening, the following fourteen persons, on applications duly certified, were elected to individual membership:

L. Bianchi. Prof., Univ. of Pisa, Pisa, Italy.
Ugo Broggi, Ph.D. (Göttingen). Prof., Buenos Aires and La Plata Univs., Buenos Aires, Arg.
C. S. Cox, A.M. (Vanderbilt). Prof., Southern Coll., Birmingham, Ala.
P. J. da Cunha. Prof., Univ. of Lisbon, Lisbon, Portugal.
Federigo Enriques. Prof., Univ. of Bologna, Bologna, Italy.
G. A. Gilbert. Prof. of physics and math., St. Ignatius Univ., San Francisco, Cal.
F. D. Murnaghan, Ph.D. (Johns Hopkins). Instr., Johns Hopkins Univ., Baltimore, Md.
H. R. Park, A.B. (Southern Univ.) Teacher, Jun. Coll., Riverside, Cal.
Arthur Pelletier. Prof. of higher alg., École Polytechnique, Montreal, Can.
Salvatore Pincherle. Prof., Univ. of Bologna, Bologna, Italy.
Susan M. Rambo, A.M. (Smith). Asst. prof., Smith Coll., Northampton, Mass.
L. H. Rice, Ph.B. (Syracuse). Instr., Tufts Coll., Tufts College, Mass.

A. V. RICHARDSON, M.A. (Cambridge). Lecturer, Bishop's Coll., Lennoxville, Quebec, Can.

A. G. WEBSTER, Ph.D. (Harvard), Sc.D. (Tufts), LL.D (Hobart). Prof. of physics, Clark Univ., Worcester, Mass.

Several of those elected come as the result of a movement for enlisting foreign members, which has been carried on the past year or more by the Committee on Membership, Professor E. R. Hedrick, chairman.

In this place should be announced a list of twenty-seven persons and three institutions elected to membership by mail vote of the Council in July, 1918:

To individual membership:

R. A. ARMS, Ph.D. (Penna.). Prof., Juniata Coll., Huntingdon, Pa.

R. N. ASHMUN, A.M. (Univ. of Wash.). Computer, Internat. Boundary Commission, Washington, D. C.

H. G. AVERS, A.B. (George Washington Univ.). Coast and Geodetic Survey, Washington, D. C.

H. E. BURTON, M.S. (Iowa). Asst., U. S. Naval Observatory, Washington, D. C.

GENEVIEVE E. COFFREY, A.M. (Univ. of Wash.). Instr. in science, High School, Mackay, Ida.

C. H. GINGRICH, Ph.D. (Chicago). Prof., Carleton Coll., Northfield, Minn.

J. M. HACKLER, Ph.B. (Chicago). Chair of math., Northeastern State Normal, Tahlequah, Okla.

EMMA E. HANTHORN, A.B. (Nebraska). Instr., State Normal, Kearney, Neb.

A. C. HICKMOTT, B.S. (Dartmouth). Statistician, Conn. Gen. Life Ins. Co., Hartford, Conn.

W. G. HUBERT, Sc.D. (New York Univ.). Instr., Coll. of City of New York, New York, N. Y.

C. B. HUGINS, B.S. in M.E. (Carnegie Inst.). Wilkinsburg, Pa.

LAURA M. LUNDIN, B.S. (Mass. Inst. of Tech.). Asst. prof., Wheaton Coll., Norton, Mass.

A. G. MONTGOMERY, A.B. (West Virginia). Instr., Concord St. Normal, Athens, W. Va.

MARY S. MOONEY, A.M. (Bellevue College). Dean of women and prof., Henderson-Brown Coll., Arkadelphia, Ark.

A. L. ONDRAK, A.B. (St. Procopius). Secy., St. Procopius Coll., Lisle, Ill.

R. E. POWELL, E.E. (Ga. Sch. of Tech.). Industr. High Sch., Columbus, Ga.

J. M. RANKIN, A.B. (Maryville Coll.). Instr., Coll. of Idaho, Caldwell, Ida.

PERCIVAL ROBERTSON, Ph.B. (Yale). Instr., The Principia, St. Louis, Mo.

H. E. RUSSELL, A.M. (Wesleyan), Sc.D. (Denver). Prof., Univ. of Denver, Denver, Col.

G. E. F. SHERWOOD, A.M. (Harvard). Asso. prof., Col. Sch. of Mines, Golden, Col.

R. K. STEWARD, C.E. (Maine). Prof. of drawing and design, Mich. Agric. Coll., East Lansing, Mich.

ELLA A. M. THORP, A.B. (Minn.). Asst. instr., Univ. of Minn., Minneapolis, Minn.
G. M. V. TRYON, Fenton, Mich.
G. P. UNSELD, A.B. (Colorado). Grad. stud., Univ. of Col., Westminster, Col.
C. B. WATTS, A.B. (Indiana). Asst., U. S. Naval Observatory, Washington, D. C.
J. J. WIDMAYER, Jr., M.S. (St. John's Coll.). Structural designer, Navy Dept., Washington, D. C.
C. C. WYLIE, A.M. (Missouri). U. S. Naval Observatory, Washington, D. C.

To institutional membership:

SOUTHWESTERN COLLEGE, Winfield, Kan.
COLLEGE OF ST. THOMAS, St. Paul, Minn.
NEW MEXICO NORMAL UNIVERSITY, East Las Vegas, N. M.

The Council transacted further business in connection with the MONTHLY and with the annual meeting of the Association. In view of the developments at the Dartmouth meeting it seemed inevitable and of the greatest importance that the subject of mathematics courses for the period of the war should form the predominating feature of the December program. Whether the Association should meet in affiliation with the American Mathematical Society, whether in affiliation with the American Association for the Advancement of Science at Baltimore, whether the Association will best serve the interests of its members by holding one meeting or, instead of this, enable more members to be within reach of the meetings by having duplicate programs in the East and in the West, were the subjects of earnest discussion. By the time this report appears it will doubtless have been decided by the full participation of the Council as to what place is wisest. W. D. CAIRNS, *Secretary-Treasurer*.

MATHEMATICAL ENCYCLOPEDIC DICTIONARY.

By G. A. MILLER, University of Illinois.

The following preliminary article is intended to serve as a basis for discussions relating to the nature and the extent of the major articles in the proposed mathematical dictionary. It aims to explain the terms *group* and *group theory* and to define a few of the most important terms which are related thereto. The latter terms should probably appear in their regular alphabetical places with references to the words *group* or *group theory* for their special meanings in this connection. In some cases this special meaning could not be made clear without such preliminary general developments as are here presented.

The object has been to give only such information as is within the range of the first-year graduate student, since the proposed dictionary should clearly not aim